# EFFECT OF THE SURFACE ON THE TECHNICAL COHESIVE STRENGTH OF SOLIDS

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The statistical strength theory, which is based on the concept of the weakest link [1], presupposes that fracture is determined by the local strength of the weakest element in the volume. However, there are data in many papers [2-6] indicating the paramount importance of surface defects for the technical cohesive strength of specimens. At the same time, a solution was obtained of the statistical problem [7] with an allowance for the surface, according to which the surface effect is significant only for solids or cross sections whose dimensions are comparable to those of the defects, while the strength depends on the volume in other cases. The surface effect was considered by means of arbitrary coefficients accounting for especially critical conditions and the density of surface imperfections.

In order to determine more specifically the effect of the surface, it is necessary to analyze the criticality of volume and surface defects and then establish a functional relationship between their parameters and the dimensions of the solid.

1. We shall start the analysis of criticality by considering cracks. In the cases of tensile stress acting on an infinitely long and a semi-infinite solid with an internal disk-shaped crack and a surface crack with the depth $x = \ell$ and the diameter $y = 2\ell$ [8], we have the following:

$$K_{\mathrm{I}} = (2/\pi)\sigma\sqrt{\pi y/2}; \tag{1.1}$$

$$K_{\mathrm{I}} = 1.12(2/\pi)\sigma\sqrt{\pi x}. \tag{1.2}$$

The cracks have equal criticality in the sense of equality of the coefficients of stress intensity if, according to (1.1) and (1.2),

$$y/x = \beta = 2\cdot 1.12^2, \tag{1.3}$$

where the first factor reflects symmetry with respect to the boundary of the solid; the second factor can be considered as a correction coefficient which accounts for the effect of the free surface; it is larger than unity due to the fact that the crack opening increases.

Evidently, by analogy with (1.3), $\beta > 2$ for other stress raisers. Actually, for a round opening and a semicircular side groove with the same radius, the stress concentration coefficients are equal to $K_t = 3$ and 3.065, respectively. Qualitatively, such a difference holds also for elliptic stress raisers [9].

As in the case of cracks, for equal values of $K_t$ of congruent stress raisers, the criticality criterion can be related to the dimensions of the latter, since, in the first place, a larger volume is exposed to the action of the larger stress raiser, and, in the second place, fracture is more likely to start there from the energy point of view because a larger amount of elastic energy is released in this case.

2. In order to determine the surface effect, we shall find the most probable dimension of the most critical defect at the surface and in the volume of the solid. The probability that the sample N contains a defect with the dimension x, while all the other imperfections are smaller than x, is given by

$$W(x) = Nf(x)\delta x[F(x)]^{N-1}$$

where f(x) and F(x) are the density and function of the defect distribution with respect to the parameter x, respectively. The maximum of W(x) is found from the condition $W'(x) = 0$ or $f'(x)F(x) + f^2(x)[N-1] = 0$.

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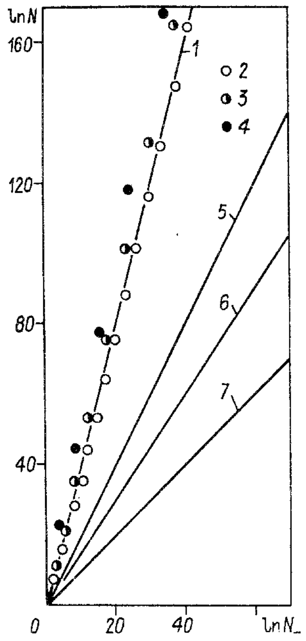


Fig. 1

Then, the sought values of the defect parameters are found from the equations

$$p'_{-}(x)\,F_{-}(x) + p_{-}^{2}(x)\,(S\,\langle n_{-}\rangle - 1) = 0; \tag{2.1}$$

$$f'(y)F(y) + f^{2}(y)\,(V\,\langle n\rangle - 1) = 0. \tag{2.2}$$

Here $p_{-}(x)$, $F_{-}(x)$, $f(y)$, and F(y) are the distribution density and function of the random quantities x and y, which determine the dimensions of defects at the surface S and in the volume V of the solid, and $\langle n_{-}\rangle$ and $\langle n\rangle$ are their densities.

For the simultaneous solution of the equations, it is necessary to establish a functional relationship between the quantities in (2.1) and (2.2). This is possible if we assume that no additional defect develops in surface forming, and only those volume imperfections which are intersected by the solid's boundary are taken into account in calculations. In this case, if we know the values of $\langle n\rangle$, f(y) and F(y) in the volume, we can readily find the same quantities for the surface.

The density of defects at the surface is

$$\langle n_{-}\rangle = \langle n\rangle \int\limits_0^\infty y f(y)\,dy = \langle n\rangle\, m \quad \left(m = \int\limits_0^\infty y f(y)\,dy \text{ is the median}\right). \tag{2.3}$$

The probability that the depth $\xi$ of a defect showing at the surface is smaller than x is

$$F_{-}(x) = \langle n_{-}\rangle^{-1} \langle n\rangle \left[\int\limits_0^x \xi f(\xi)\,d\xi + \int\limits_x^\infty x f(\xi)\,d\xi\right].$$

After performing some transformations, we obtain

$$F_{-}(x) = m^{-1} \int\limits_0^x \xi f(\xi)\,d\xi + x\,[1 - F(x)]/m. \tag{2.4}$$

The distribution density of x is

$$p_{-}(x) = F'_{-}(x) = [1 - F(x)]/m; \tag{2.5}$$

$$p'_{-}(x) = -f(x)/m. \tag{2.6}$$

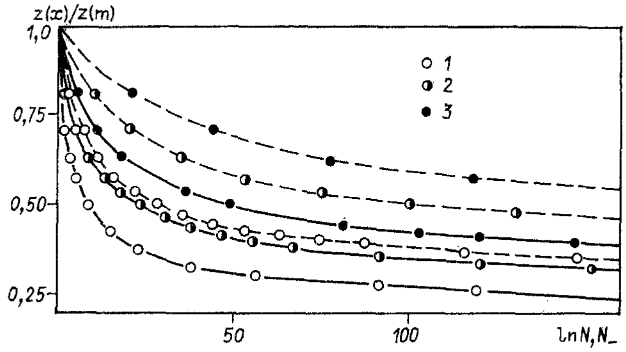


Fig. 2

Assume that y is characterized by the distribution law

$$f(y) = c\exp[-\alpha(y-m)^2], \quad \int_0^\infty f(y)\,dy = 1 \tag{2.7}$$

(m and α are the distribution constants). Then, after the substitution of (2.3)-(2.6) with an allowance for (2.7), we obtain the following for $x > m + 3\sigma$ $(F(y) \approx 1 - [2\alpha(y-m)^{-1}f(y)$ [10]) after simple transformations:

$$\exp\,[-\alpha(x-m)^2] = 4\alpha^2 m\,(x-m)^2/c(N_- - 1). \tag{2.8}$$

The equation for volume defects, which is similar to (2.8), is given by

$$\exp\,[-\alpha(y-m)^2] = 2\alpha(y-m)/cN. \tag{2.9}$$

If we add condition (1.3) to Eqs. (2.8) and (2.9),

$$y = \beta x, \tag{2.10}$$

then, simultaneous solution yields the relationship between the solid's volume and surface with defects of equal criticality.

With an allowance for 2.10, we transform (2.8) and (2.9), reducing them to the following:

$$\alpha x^2 - [2\alpha m x - \alpha m^2 - \ln\alpha(x-m)^2] = \ln\,(cN_-/4\alpha m); \tag{2.11}$$

$$\alpha\beta^2x^2 - [2\alpha\beta x m - \alpha m^2 - \ln\sqrt{\alpha}(y-m)] = \ln\,(cN/2\sqrt{\alpha}). \tag{2.12}$$

We can simplify this system considerably by neglecting the terms within square brackets on the left-hand sides (this assumption will be verified below):

$$\alpha x^2 = \ln\,(N_-\,c/4\alpha m),\ \alpha\beta^2 x^2 = \ln\,(Nc/2\sqrt{\alpha}). \tag{2.13}$$

Dividing the first equation by the second one, we obtain

$$N = c_1\,(N_-)^{\beta^2}. \tag{2.14}$$

Expression (2.14) signifies that, if equal criticality is to apply to the surface and the volume, the latter must increase in proportion to $L^{\beta^2}$ as the surface increases in proportion to L, if L is the characteristic dimension of the solid. Since $\beta^2 > 4$, this is virtually impossible.

Actually, in an $N_-$ vs. N plot, Eq. (2.14) constitutes the boundary (Fig. 1) dividing the region into two parts, the top one pertaining to the effect of the volume, and the bottom one to the effect of the surface. Curves 5-7 reflect the most characteristic laws of variation of the solid's volume and surface: $L^2$ and L, $L^3$ and $L^2$, and L and L. They are all located below curve 1 in the region of surface defects. Such a boundary, plotted on the basis of the conclusions reached in [7], passes along curve 7, which indicates that the surface effect has been underestimated considerably.

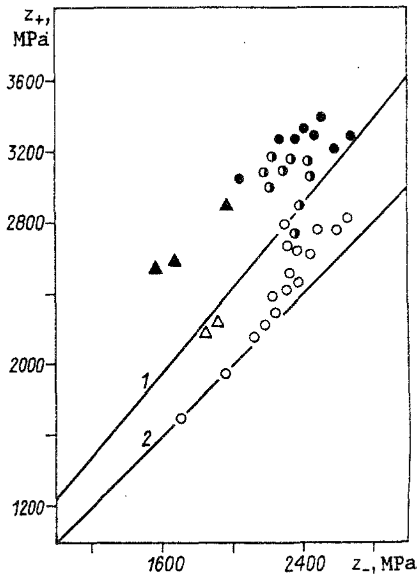


Fig. 3

The approximation (2.13) was checked for different values of $\sigma = (2\alpha)^{-1/2}$ for m = 1 (points 2-4 for $\sigma$ = 1; 0.5; 0.33) in solving numerically Eqs. (2.11) and (2.12) on a computer.

Figure 2 shows the same calculation results, given in a ln (N, N₋) vs $z(x)/z(m)$ $(z(x)$ plot (z(x) is the strength of a solid containing a crack whose dimension is x: $z(x)/z(m) = (x/m)^{-1/2}$) ; the dashed curves pertain to fracture developing in the volume, while the solid curves correspond to fracture initiated at the surface; the points 1-3 denote $\sigma$ = 1; 0.5; 0.33.

Relationship (2.14) makes it possible to find the mean number of fractures developing from the surface $\gamma$ for each instance of fracture initiated in the volume. For a cylindrical specimen whose diameter and length are equal to D, the numbers of defects within the volume and at the side surface are equal to $\langle n\rangle \pi D^3/4$ and $\langle n\rangle m\pi D^2$ , respectively. Neglecting the surface layer volume, we write the equal-strength condition (2.14) as follows:

$$\gamma \langle n\rangle \pi D^3/4 = c_1 (\langle n\rangle m\pi D^2)^{\beta^2};$$

$$\gamma = c_2 (D/m)^{2\beta^2-3} \quad (c_2 = 4c(\pi \langle n\rangle)^{\beta^2-1} m^{3(\beta^2-1)}).$$

For the extreme case $\beta$ = 2 and for n = 1, m = 1, and $\alpha$ = 1,

$$\gamma \approx 0.7\cdot 10^2 (D/m)^5.$$

For example, if D ~ 10 m, we have $\gamma \sim 10^7$.

Neutralization of surface defects should increase the strength by a factor of $\eta = z(y^*)/z(x^*)$ on the average (y* and x* are the sought dimensions of defects within the volume and at the surface). In the case of cracks, we have $\eta = (\beta x^*/y^*)^{1/2}$ in accordance with (1.1) and (1.2).

The effect of the surface has been indicated above for a distribution f(y) which has a maximum. For instance, such a distribution is characteristic for window glass panes; however, a peak is not always present [11]. In connection with this, a similar analysis was performed for $f(y) = (2/\sigma\sqrt{2\pi}) \exp[-y^2/2\sigma^2]$ , which led to a result similar to (2.14).

3. Experimental confirmation of the role of the surface is given in [3-6]. It was found there that the strength is considerably enhanced by neutralizing surface defects in materials which are in a brittle state. However, the test results indicate that the surface effect also extends to the mechanical characteristics of steel in a high-strength state during tensile tests.

The test specimens, made of 40KhN steel, with a diameter of 10 mm and a length of 100 mm, were subjected to isothermic hardening at 850°C and annealed in saltpeter at 200°C over a period of 30 min ($\sigma_{ult}$ = 2190 MPa). They were subjected to tensile tests and brought to failure after plastic straining with necking as well as in a quasi-brittlement manner, i.e., with slight elongation and without local reduction in area. In the absence of the neck, when the stress was uniform over the cross section, all specimens failed by fracture progressing from the surface. The specimen displaying contraction failed by fracture initiated at the surface as well as by rupture at the middle of the neck.

Since the distribution of the axial stress is not uniform over the cross section of the neck, as it displays a maximum at the center and depends on the neck dimensions, we can determine the moment at which central failure replaces surface failure.

The stresses at the surface and at the neck center are analyzed by means of the equations [12]

$$z_+ = z(R + 0.5a)/(R + 0.25a), \; z_- = zR/(R + 0.25a),$$

where $z_+$, z and $z_-$ are the central, the mean nominal, and the surface values of the axial stress over the section, a is the cross-sectional radius of the neck, and R is the curvature radius of its contour in the meridional cross section. The latter is found by means of a tool microscope, applying templates with different curvatures.

The test results have shown (Fig. 3) that, in all cases (18 of 30), fracture is initiated at the surface (open circles) when $z_+/z_- < 1.23$ (between curves 1 and 2), and it is initiated at the middle of the neck when $z_+/z_- > 1.23$ (filled circles; this also applies to triangles for specimens annealed at 300°C), i.e., $\eta = 1.23$, $x^* \approx 0.5y^*$ according to (1.3).

For neutralizing the surface defects, softening of the surface layer of specimens to a depth of 0.5 mm was effected by means of induction annealing. As a result, failure initiated at the surface was completely eliminated (for more than 100 specimens; half-filled circles in Fig. 3). The mean strength at fracture was thus increased from 2590 to 2750 MPa in spite of the softening of a part of the cross section.

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